

$$2z^2 + 3z + 7 = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-3 \pm \sqrt{9 - 66}}{4}$$

$$z = \frac{-3 \pm \sqrt{-47}}{4} = \frac{-3 \pm \sqrt{47}j}{4}$$

Roots of General Polynomials :

Every polynomial degree  $n$  has  $n$  roots.

$$(x-1)^2 = 0$$

$x=1$  , but this is repeated root

If polynomial has only real coefficients then roots occur in conjugate pairs.

For cubic with real coefficients, it must have at least one real root as it will have complex conjugate pair and real root or 3 real roots.

For quartic, it may have 4 complex roots - therefore never cross the  $x$ -axis (2 complex conjugates).

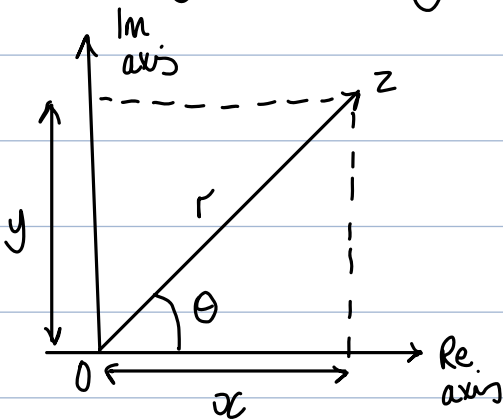
Why does complex number division work?

Because if  $z = x + jy$

$$z\bar{z} = x^2 + y^2 = |z|^2$$

← Modulus (absolute value)

The Argand Diagram: (Complex Plane)



$$x = r \cos \theta$$

$$y = r \sin \theta$$

$r, \theta$  real

$$\therefore z = x + jy$$

$$z = r \cos \theta + j (r \sin \theta)$$

$$z = r (\cos \theta + j \sin \theta)$$

Polar form

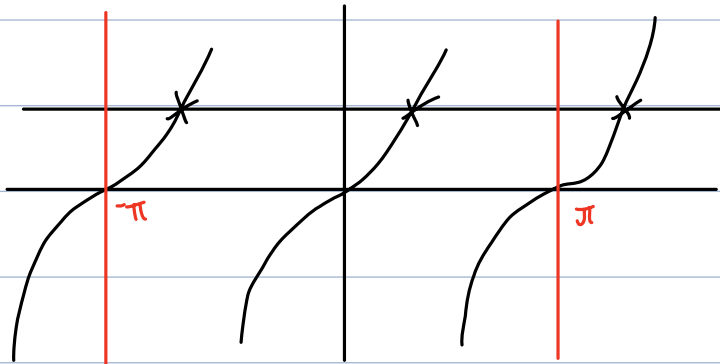
↑  $|z|$  - modulus / length of  $z$

$$r = \sqrt{x^2 + y^2} \geq 0$$

$\theta$  is argument (or phase) of  $z$

$$\theta = \arg z = \tan^{-1} \left( \frac{y}{x} \right)$$

Which arctan value do you use?

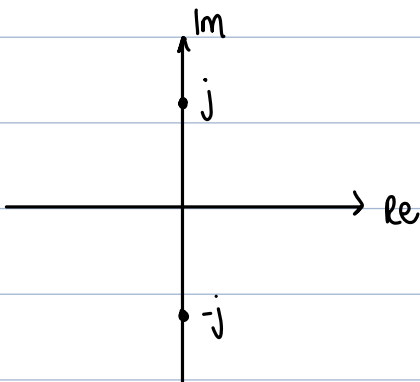


Convention is we take answers between  $-\pi$  and  $\pi$ .

$$\theta \in (-\pi, \pi]$$

'includes'  $\leftarrow$   $-\pi$   $\leftarrow$   $\pi$  includes  $\pi$   
 $\leftarrow$  excludes  $-\pi$

aka  $-\pi < \theta \leq \pi$



Quickest way to get polar form of numbers is to draw Argand Diagram.

Complex Conjugate - reflect in real axis:

$$\text{Arg}(\bar{z}) = -\text{Arg}(z)$$

$$z = x + jy$$

$$z^* = x - jy$$

conjugate

$$z + z^* = 2x = 2\operatorname{Re}(z)$$

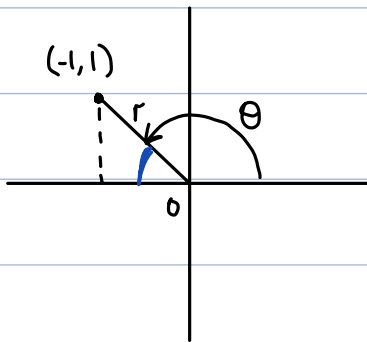
$$z - z^* = 2jy = 2\operatorname{Im}(z)$$

$$zz^* = x^2 + y^2$$

$$(z_1 z_2)^* = z_1^* z_2^*$$

When finding  $|z|$  it's  $\sqrt{x^2 + y^2}$   
NOT  $jy$

Draw  $\theta$  in the right quadrant.



Angle given from trig value  
 $-\tan\left(\frac{1}{1}\right) = \frac{1}{4}\pi$

Must do  $\pi - \frac{\pi}{4}$  for  $\theta$